Sample Question Paper - 27
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed : 2 hours *Maximum Marks : 40*

General Instructions :

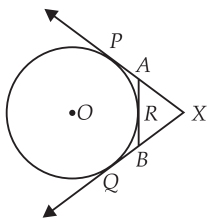
1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *All questions are compulsory.*
3. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
4. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
5. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

SECTION - A

1. The mean of marks in Mathematics for 40 students in a class was 56.5. Later it was found that the marks given to one student was incorrectly entered as 85 instead of 58. Find the correct mean for the class.

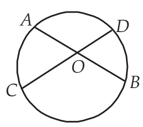
2. From the top of a cliff 20 m high, the angle of elevation of the top of a tower is found to be equal to the angle of depression of the foot of the tower. Find the height of the tower.

3. In figure, XP and XQ are tangents from X to the circle with centre O. R is a point on the circle. Prove that, $XA + AR = XB + BR$.



OR

In given fig., two chords AB and CD of a circle intersect at O. If $AO = 8$ cm, $CO = 6$ cm and $OD = 4$ cm, then find the length of OB.



4. The string of a kite is 120 m long and it makes an angle of 60° with the horizontal. Find the height of the kite assuming that there is no slack in the string. [Use $\sqrt{3} = 1.732$]

5. A solid is in the shape of a cone mounted on a hemisphere of same base radius. If the curved surface areas of the hemispherical part and the conical part are equal, then find the ratio of the radius and the height of the conical part.

OR

Two cubes each of 10 cm edge are joined end to end. Find the surface area of the resulting cuboid.

6. Find the mean of following distribution:

x_i	4	6	9	10	15
f_i	5	10	10	7	8

SECTION - B

7. If the sum of n terms of an A.P. is given by $S_n = (3n^2 + 2n)$, find its (i) n^{th} term (ii) first term (iii) common difference.

8. Draw a line segment of length 7 cm and divide it internally in the ratio 2 : 3.

9. The digits of a positive number of three digits are in A.P. and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number.

OR

In an A.P., $S_m = n$ and $S_n = m$ also $m > n$, find the sum of first $(m - n)$ terms.

10. The shadow of a tower standing on a leveled ground is found to be 30 m longer when the sun's altitude is 30° than when it is 60°? Find the height of the tower.

SECTION - C

11. A train travels at a certain average speed for a distance of 63 km and then travels a distance of 72 km at an average speed of 6 km/hr more than its original speed. If it takes 3 hours to complete total journey, what is the original average speed?

OR

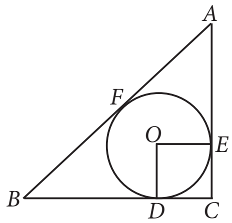
The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

12. From a solid cylinder whose height is 15 cm and diameter 16 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid. [Take $\pi = 3.14$]

Case Study - 1

13. Raghav loves geometry. So he was curious to know more about the concepts of circle. His father is a mathematician. So, he reached to his father to learn something interesting about tangents and circles. His father gave him knowledge on circles and tangents and ask him to solve the following questions.

(i) A circle of radius 3 cm is inscribed in a right angled triangle BAC such that $BD = 9$ cm and $DC = 3$ cm. Find the length of AB.



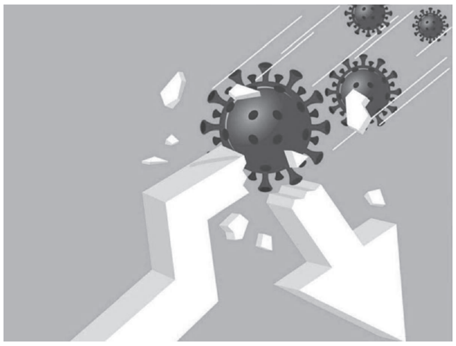
(ii) If PA and PB are two tangents to a circle with centre O from an external point P such that $\angle OPB = 40°$, then find the value of $\angle BPA$.

Case Study - 2

14. Household income in India was drastically impacted due to the COVID-19 lockdown. Most of the companies decided to bring down the salaries of the employees by 50%.

The following table shows the salaries (in percent) received by 25 employees during lockdown.

Salaries received (in percent)	50-60	60-70	70-80	80-90
Number of employees	9	6	8	2



Based on the above information, answer the following questions.

(i) Find the median class of the given data.

(ii) If x_i's denotes the class marks and f_i's denotes the corresponding frequencies for the given data, then find $\Sigma x_i f_i$.

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

1. Number of students in the class(n) = 40

Now, mean $(\bar{x}) = \frac{\sum x}{n}$

$\Rightarrow 56.5 = \frac{\sum x}{40}$ [Given]

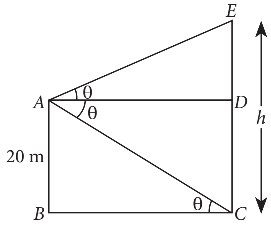
$\Rightarrow \sum x = 56.5 \times 40 \quad \Rightarrow \sum x = 2260$

But 2260 is incorrect sum.

Since, correct sum = 2260 – 85 + 58 = 2233

$\therefore$ Correct mean = $\frac{2233}{40} = 55.82$

2. Let AB be the cliff and $EC = h$ m be the height of tower.



$\therefore$ $ED = EC - DC = EC - AB$

$= (h - 20)$m [$\because DC = AB$]

In ΔABC, $\tan\theta = \frac{AB}{BC} = \frac{20}{BC}$

$\Rightarrow BC = \frac{20}{\tan\theta}$ ...(i)

In ΔEAD,

$\tan\theta = \frac{ED}{AD} = \frac{ED}{BC} = \frac{(h-20)\tan\theta}{20}$ [(Using (i)]

$\Rightarrow h - 20 = 20 \quad \Rightarrow \quad h = 40$

$\therefore$ Height of tower is 40 m.

3. Since, length of tangents from an exterior point to a circle are equal.

$\therefore$ $XP = XQ$ [Tangents from X] ...(i)

$AP = AR$[Tangents from A] ...(ii)

$BQ = BR$ [Tangents from B] ...(iii)

Now, $XP = XQ$

$\Rightarrow XA + AP = XB + BQ$

$\Rightarrow XA + AR = XB + BR$ [Using (ii) and (iii)]

OR

We have, OA = 8 cm, OC = 6 cm, OD = 4 cm

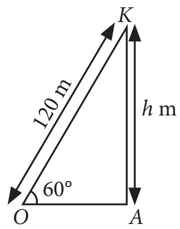
Since, AB and CD are chords of the circle intersect at O.

$\therefore$ $OA \times OB = OC \times OD$

$\Rightarrow 8 \times OB = 6 \times 4$

$\Rightarrow OB = 3$ cm

4. Let OA be the horizontal ground and K be the position of the kite at a height h m above the ground. Then, $AK = h$ m, OK = 120 m and $\angle AOK = 60°$



In ΔAOK, $\sin 60° = \frac{AK}{OK}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{h}{120}$

$\Rightarrow h = 120 \times \frac{\sqrt{3}}{2} = 60\sqrt{3}$

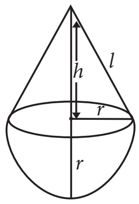
$\Rightarrow h = 60 \times 1.732 = 103.92$

Hence, the height of the kite is 103.92 m.

5. Let r be the radius of cone or hemispherical part and h be the height of cone.

According to question,

$2\pi r^2 = \pi rl$, where l is slant height of cone



$\Rightarrow 2r = l \quad \Rightarrow \quad 4r^2 = l^2$

$\Rightarrow 4r^2 = r^2 + h^2$

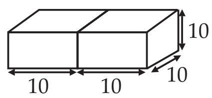
$\Rightarrow 3r^2 = h^2$

$\Rightarrow \frac{r}{h} = \frac{1}{\sqrt{3}}$

$\therefore$ Required ratio = $1 : \sqrt{3}$

OR

If two cubes are joined end to end, we get a cuboid such that



l = Length of the resulting cuboid

= 10 cm + 10 cm = 20 cm

b = Breadth of the resulting cuboid = 10 cm

h = Height of the resulting cuboid = 10 cm

$\therefore$ Surface area of the cuboid = $2(lb + bh + lh)$

$\Rightarrow$ Surface area of the cuboid

$= 2(20 \times 10 + 10 \times 10 + 20 \times 10)$ cm^2

$\therefore$ Surface area of the cuboid is 1000 cm^2

6. Let us construct the following table for the given data:

x_i	f_i	$f_i x_i$
4	5	20
6	10	60
9	10	90
10	7	70
15	8	120
Total	$\Sigma f_i = 40$	$\Sigma f_i x_i = 360$

$$\therefore \text{Mean}(\bar{x}) = \frac{\sum_{i=1}^{5} x_i f_i}{\sum_{i=1}^{5} f_i} = \frac{360}{40} = 9$$

7. Given, $S_n = (3n^2 + 2n)$

$\therefore$ $S_{n-1} = \{3(n-1)^2 + 2(n-1)\}$

$= (3n^2 - 4n + 1)$

(i) The n^{th} term is given by

$T_n = (S_n - S_{n-1}) = \{(3n^2 + 2n) - (3n^2 - 4n + 1)\} = (6n - 1)$

$\therefore$ n^{th} term $= (6n - 1)$ ...(1)

(ii) Putting $n = 1$ in (1), we get

$T_1 = (6 \times 1 - 1) = 5$

$\therefore$ First term = 5

(iii) Putting $n = 2$ in (1), we get

$T_2 = (6 \times 2 - 1) = 11$

$\therefore$ $d = T_2 - T_1 = 11 - 5 = 6.$

8. Steps of construction:

Step-I : Draw a line segment $AB = 7$ cm.

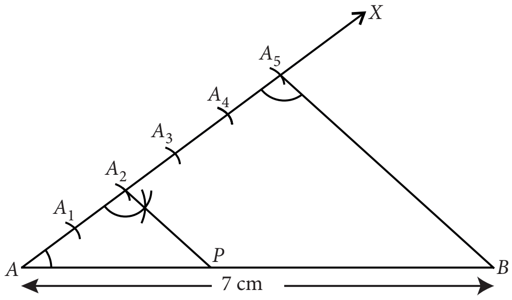
Step-II : Draw any ray AX making an acute angle with AB.

Step-III : On ray AX, locate 5(= 2 + 3) points A_1, A_2, A_3, A_4, A_5 such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$.

Step-IV : Join A_5B.

Step-V : From A_2, draw a line parallel to A_5B, intersecting AB at P.

Thus, P divides AB in the ratio 2 : 3.



9. Let the required three digit number be xyz.

$\because$ Digits are in A.P.

$\therefore$ $x = y - d$ and $z = y + d$ where d is common difference

According to question,

$(y - d) + y + (y + d) = 15$

$\Rightarrow 3y = 15 \Rightarrow y = 5$

Since, the number obtained by reversing the digits $(z\,y\,x)$ *i.e.*, $100z + 10y + x$ is 594 less than original number.

$\therefore$ $(100x + 10y + z) - (100z + 10y + x) = 594$

$\Rightarrow (z - 100z) + (100x - x) = 594$

$\Rightarrow 99x - 99z = 594$

$\Rightarrow x - z = 6$

$\Rightarrow (y - d) - (y + d) = 6$

$\Rightarrow -2d = 6 \Rightarrow d = -3$

So, $x = y - d = 5 - (-3) = 8$ and $z = y + d = 5 - 3 = 2$

$\therefore$ The number is xyz or 852.

OR

Let a, $a + d$, $a + 2d$, ... be the A.P.

Given, $S_m = \frac{m}{2}[2a + (m-1)d] = n$

$\Rightarrow 2a + (m-1)d = \frac{2n}{m}$ ...(1)

And $S_n = \frac{n}{2}[2a + (n-1)d] = m$

$\Rightarrow 2a + (n-1)d = \frac{2m}{n}$ ...(2)

Subtracting (2) from (1), we get

$2a + (m-1)d - 2a - (n-1)d = \frac{2n}{m} - \frac{2m}{n}$

$\Rightarrow (m - 1 - n + 1)d = \frac{2n^2 - 2m^2}{mn}$

$\Rightarrow d = -\frac{2(m+n)}{mn}$ ...(3)

Now, $S_{m-n} = \frac{m-n}{2}\{2a + (m-n-1)d\}$

$= \frac{m-n}{2}\{2a + (m-1)d - nd\}$

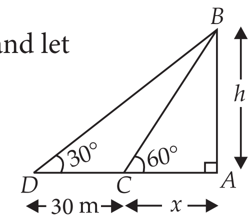
$= \frac{m-n}{2}\left\{\frac{2n}{m} + \frac{2(m+n)}{m}\right\}$ [Using (1) & (3)]

$= \frac{m-n}{2}\left\{\frac{4n+2m}{m}\right\} = \frac{(m-n)(2n+m)}{m}$

10. Let AB be the tower and AC and AD be its shadows when the angles of elevation are 60° and 30° respectively.

Then $CD = 30$ m.

Let h be the height of the tower and let $AC = x$ m.



In ΔABC, right angled at A

$\tan 60° = \frac{AB}{AC}$

$\Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow x = \frac{h}{\sqrt{3}}$ ...(1)

In ΔDAB, we have

$\tan 30° = \frac{AB}{AD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+30} \Rightarrow x + 30 = \sqrt{3}h$ ...(2)

Putting value of x from (1) in (2), we get

$\frac{h}{\sqrt{3}} + 30 = \sqrt{3}h$

$\Rightarrow\ h+30\sqrt{3}=3h \Rightarrow 2h=30\sqrt{3} \Rightarrow h=15\sqrt{3}$

Thus, the height of the tower is $15\sqrt{3}$ m.

11. Let original average speed of the train be x km/hr.

According to question,

$$\frac{63}{x}+\frac{72}{x+6}=3$$

$$\Rightarrow \frac{7}{x}+\frac{8}{x+6}=\frac{1}{3} \Rightarrow \frac{7(x+6)+8x}{x(x+6)}=\frac{1}{3}$$

$\Rightarrow 3(7x + 42 + 8x) = x^2 + 6x$

$\Rightarrow 45x + 126 = x^2 + 6x \Rightarrow x^2 - 39x - 126 = 0$

$\Rightarrow x^2 - 42x + 3x - 126 = 0 \Rightarrow (x - 42)(x + 3) = 0$

$\Rightarrow$ Either $x = 42$ or $x = -3$

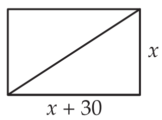
$\Rightarrow x = 42\ (\because x > 0)$

Hence, the original speed of the train is 42 km/h.

OR

Let the shorter side (*i.e.*, breadth) = x m.

$\therefore$ The longer side (length) = $(x + 30)$ m.



In a rectangle,

diagonal $=\sqrt{(\text{breadth})^2+(\text{length})^2}$

$$\Rightarrow x+60=\sqrt{x^2+(x+30)^2}$$

$$\Rightarrow x+60=\sqrt{x^2+x^2+60x+900}$$

$\Rightarrow (x + 60)^2 = 2x^2 + 60x + 900$ [Squaring both sides]

$\Rightarrow x^2 + 120x + 3600 = 2x^2 + 60x + 900$

$\Rightarrow x^2 - 60x - 2700 = 0$

$$\Rightarrow x=\frac{-(-60)\pm\sqrt{14400}}{2(1)} \Rightarrow x=\frac{60\pm120}{2}=90,-30$$

As, $x \neq -30 \Rightarrow x = 90$ [$\because$ breadth cannot be negative]

$\therefore$ The longer side = $x + 30 = 90 + 30 = 120$

Thus, the shorter side = 90 m.

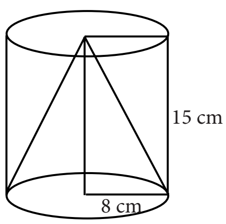
The longer side = 120 m.

12. Height of cylinder,

h = 15 cm

Radius of cylinder,

$r=\frac{16}{2}=8$ cm



Height of cone, h = 15 cm

Radius of the cone, r = 8 cm

Slant height of the cone, $l=\sqrt{8^2+15^2}$

$=\sqrt{64+225}=\sqrt{289}=17$ cm

$\therefore$ Curved surface area of the cone = πrl

$= 3.14 \times 8 \times 17 = 427.04\ \text{cm}^2$

Curved surface area of the cylinder = $2\pi rh$

$= 2 \times 3.14 \times 8 \times 15 = 753.6\ \text{cm}^2$

Area of the top face of the cylinder = πr^2

$= 3.14 \times (8)^2 = 200.96\ \text{cm}^2$

$\therefore$ Total surface area of the remaining solid

= Area of the top face of the cylinder + curved surface area of the cylinder + curved surface area of the cone

$= 200.96 + 753.6 + 427.04 = 1381.6\ \text{cm}^2$

13. (i) Let $AF = AE = x$ cm

[$\because$ Tangents drawn from an external point to a circle are equal in length]

$\therefore$ $BD = FB = 9$ cm, $CD = CE = 3$ cm

In ΔABC, $AB^2 = AC^2 + BC^2$

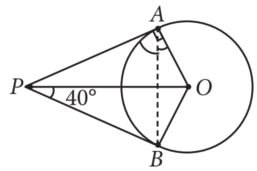
$\Rightarrow (AF + FB)^2 = (AE + EC)^2 + (BD + CD)^2$

$\Rightarrow (x+9)^2=(x+3)^2+(12)^2 \Rightarrow 18x+81=6x+9+144$

$\Rightarrow 12x = 72 \Rightarrow x = 6$ cm

$\therefore$ $AB = 6 + 9 = 15$ cm

(ii) Here, $\angle OAP = 90°$



In ΔAOP and ΔBOP,

$\angle OAP = \angle OBP = 90°$

$OA = OB$ [Radii of circle]

$PA = PB$ [Tangents drawn from an external point are equal]

$\therefore$ $\Delta AOP \cong \Delta BOP$ [By SAS congruency]

$\therefore$ $\angle APO = \angle OPB = 40°$ [By C.P.C.T.]

$\therefore$ $\angle BPA = 40° + 40° = 80°$

14. (i) The cumulative frequency distribution table for the given data can be drawn as :

Salaries received (in percent)	Number of employees (f_i)	Cumulative frequency (*c.f.*)
50-60	9	9
60-70	6	9 + 6 = 15
70-80	8	15 + 8 = 23
80-90	2	23 + 2 = 25
Total	$\Sigma f_i = 25$	

Here, $\frac{N}{2}=\frac{25}{2}=12.5$

The cumulative frequency just greater than 12.5 lies in the interval 60-70.

Hence, the median class is 60-70.

(ii) Let us consider the following table :

Class	Class mark (x_i)	Frequency (f_i)	$x_i f_i$
50-60	55	9	495
60-70	65	6	390
70-80	75	8	600
80-90	85	2	170
Total		$\Sigma f_i = 25$	$\Sigma x_i f_i = 1,655$